Simulation of the Brownian motion

Exercise 1 *Simulate* 10 *paths of the Ornstein-Uhlenbeck process on the time interval* $[0,1]$ *with step size* $\Delta t = 0.02$
We recall that the Orstein-Uhlenbeck process is a continuous gaussian process $(X_t,\, t \geq 0)$ *satisfying* $\mathbb{E}[X_t] = 0$ *for all* $t \geq 0$ *and* $\mathbb{E}[X_t X_s] = \exp(-\gamma|t-s|)$, γ *being a positive constant. We also assume that* $X_0 = 0$.

Exercise 2 *The fractional Brownian motion is a continuous gaussian process* $(B^H(t),\, t \geq 0)$ *starting in* 0 *such that* $\mathbb{E}[B_t^H] = 0$ *for all* $t \geq 0$ *and* $\mathbb{E}[B_t^H B_s^H] = \frac{1}{2}\left(t^{2H} + s^{2H} - |t-s|^{2H}\right)$. *The parameter* H *is called the Hurst index.*

1. *Verify that the fractional Brownian motion is a Brownian motion when the parameter satisfies* $H = 1/2$.
2. *Simulate one trajectory of this process on the time interval* $[0,1]$ *and with the step size* $\Delta t = 0.02$.

Exercise 3 *Let* Q *the following covariance matrix:*

$$Q = \begin{pmatrix} 1 & 1 & -0.8 \\ 1 & 1.04 & -0.8 \\ -0.8 & -0.8 & 0.68 \end{pmatrix}.$$

Let (B_t) *be a 3-dimensional* Q*-Brownian motion. Represent on the same figure, a simulation of three trajectories each corresponding on one coordinate of* (B_t). *Comment the dependences between the different coordinates of the* Q*-Brownian motion (interval* $[0,1]$ *and step size* $\Delta t = 0.01$*).*